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Comparison of fuzzy numbers using close interval distance

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Abstract

A new distance measure called 'close interval distance' associated with interval approximation of a fuzzy number is proposed. A distinct approach for ranking fuzzy numbers with the aid of this measure is introduced. Also some more results related on properties of ordering using this measure are obtained

Keywords: close interval approximation, distance measure, fuzzy numbers, ordering, linear order

1. INTRODUCTION

The purpose of this paper is to propose a new method for ranking fuzzy numbers. Gonzalez (1990) used average value to define ordering. Saade and Schwarzlander (1992) used interval to consider ordering. Their definition was more complicated and they used non-negative values to compare the ordering of fuzzy numbers. In this paper we use close interval distance associated with interval approximation of fuzzy numbers, to define ordering and we use both positive and negative values to define ordering. Yager (1981) used a weighted mean value to define ordering.

In this paper, let F be the family of the fuzzy numbers on R. We define this distance measure with the help of □-cuts We shall prove that the ordering relation $\prec$, $\approx$ on F satisfy the law of trichotomy. Some more results connected to ordering properties are studied with the help of close interval distance measure. In general this ranking method possesses several good characteristics and advantages as compared to other existing ranking methods.

2. BASICS

2.1. Definition

A fuzzy subset A of the real line R with membership. Function

μ_A: R $\rightarrow$ [0,1] is called a fuzzy number if

i. A is normal
ii. A is fuzzy convex
(i.e.,) $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$
$\forall\ x_1, x_2 \in R$ and $\lambda \in [0,1]$
iii. μ_A is upper semi continuous and
iv. Supp. A is bounded.

2.2. α-Cut

An α-cut of a fuzzy set $\tilde{A}$ is a crisp set A_α that contains all the elements of the universal set X that have a membership grade greater than or equal to the specified value of α. Thus, $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$, $0 \leq \alpha \leq 1$.

2.3. Close interval approximation [4]

An interval approximation $[a_\alpha^L, a_\alpha^U]$ of fuzzy number $\tilde{A}$ is said to be close interval approximation if

$a_\alpha^L = \inf\{ x \in R \mid \mu_A(x) \geq 0.5]$ and
$a_\alpha^L = \sup\{ x \in R \mid \mu_A(x) \geq 0.5]$ and is denoted by $[A] = [a_\alpha^L, a_\alpha^U]$

2.4. α-Cuts for Different Fuzzy numbers

α-cut or interval of confidence :

For triangular fuzzy number
$[a_1+(a_2-a_1)\alpha,\ a_3-(a_3-a_2)\alpha]$

For Trapezoidal fuzzy number
$[a_1+(a_2-a_1)\alpha,\ a_4-(a_4-a_3)\alpha]$

For piecewise– quadratic fuzzy number $[a_1+(a_3-a_1)\alpha,\ a_5-(a_5-a_3)\alpha]$

2.5. Zero Fuzzy Number

The zero fuzzy number is denoted by $\hat{0}$ whose both lower and upper bounds of α-cuts ($0 \leq \alpha \leq 1$) are 0.

3. CLOSE INTERVAL DISTANCE MEASURE

3.1. Definition

Let F be a set of all fuzzy numbers. For $\tilde{A}$, $\tilde{B} \in$ F, define the close interval distance (CID) of $\tilde{A}$ and $\tilde{B}$ as follows.

$$D(\tilde{A}, \tilde{B}) = \frac{1}{2}[a_\alpha^L + a_\alpha^U - b_\alpha^L - b_\alpha^U],$$

where $[A] = [a_\alpha^L, a_\alpha^U]$ and $[B] = [b_\alpha^L, b_\alpha^U]$ are the close interval approximations of fuzzy numbers. Also $D(\tilde{A}, \tilde{B})$ means the distance of $\tilde{B}$ to $\tilde{A}$.

3.2. Results

The CID of the fuzzy number is the corresponding defuzzyfied real number.

i. For the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$,

$$D(\tilde{A}, \hat{0}) = \frac{1}{2}\left[a_1 + (a_2 - a_1)\left(\frac{1}{2}\right) + a_3 - (a_3 - a_2)\left(\frac{1}{2}\right)\right]$$

$$= \left(\frac{1}{4}\right)[a_1 + 2a_2 + a_3]$$

= defuzzified real number of the triangular fuzzy number.

ii. For the trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$,

$$D(\tilde{A}, \hat{0}) = \frac{1}{2}\left[a_1 + (a_2 - a_1)\left(\frac{1}{2}\right) + a_4 - (a_4 - a_3)\left(\frac{1}{2}\right)\right]$$

$$= \left(\frac{1}{4}\right)[a_1 + a_2 + a_3 + a_4]$$

= defuzzified real number of the trapezoidal fuzzy number.

iii. For Piecewise Quadratic Fuzzy Number (PQFN) $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$

$$D(\tilde{A}, \hat{0}) = \frac{1}{2}\left[a_1 + (a_3 - a_1)\left(\frac{1}{2}\right) + a_5 - (a_5 - a_3)\left(\frac{1}{2}\right)\right]$$

$$= \left(\frac{1}{4}\right)[a_1 + 2a_3 + a_5]$$

= defuzzified real number of the PQFN

3.3. Remark

It is noted that $D(\tilde{A}, \tilde{B}) = D(\tilde{A}, \hat{0}) - D(\tilde{B}, \hat{0})$

3.4. Definition

For $\tilde{A}$, $\tilde{B} \in$ F, Define the ranking of $\tilde{A}$, $\tilde{B}$ by saying.

i. $D(\tilde{A}, \tilde{B}) > 0$ iff $D(\tilde{A}, \hat{0}) > D(\tilde{B}, \hat{0})$ iff $\tilde{B} \prec \tilde{A}$
ii. $D(\tilde{A}, \tilde{B}) < 0$ iff $D(\tilde{A}, \hat{0}) < D(\tilde{B}, \hat{0})$ iff $\tilde{A} \prec \tilde{B}$
iii. $D(\tilde{A}, \tilde{B}) = 0$ iff $D(\tilde{A}, \hat{0}) > D(\tilde{B}, \hat{0})$ iff $\tilde{A} \approx \tilde{B}$

3.5. Examples

i. Consider two PQFNS $\tilde{A} = (1,2,3,4,5)$ $\tilde{B} = (1,3,4,5,7)$

$$D(\tilde{A}, \tilde{B}) = \frac{1}{2}[A_\alpha^L + A_\alpha^U - B_\alpha^L - B_\alpha^U]$$

$$= \frac{1}{2}[2 + 4 - 3 - 5] = -1 < 0$$

$$\Rightarrow \tilde{A} \prec \tilde{B} \text{ and } D(\tilde{A}, \hat{0}) < D(\tilde{B}, \hat{0})$$

ii. Consider two PQFNS $\tilde{A} = (2, 3, 4, 5, 6)$ and $\tilde{B} = (1,2,4,6,7)$

$$\text{Now } D(\tilde{A}, \tilde{B}) = \frac{1}{2}[3+5-2-6] = 0$$

$$\Rightarrow \tilde{A} \approx \tilde{B}$$

iii. Consider two trapezoidal fuzzy numbers $\tilde{A} = (3\ 4\ 6\ 7)$ $\tilde{B} = (2\ 4\ 6\ 8)$

The lower and upper bounds using α- cuts are

$$A_L(\alpha) = 3 + \alpha,\ A_U(\alpha) = 7 - \alpha$$

$$B_L(\alpha) = 2 + 2\alpha,\ B_U(\alpha) = 8 - 2\alpha$$

$$\therefore A_\alpha^L = 3\frac{1}{2},\quad A_\alpha^U = 6\frac{1}{2}$$

$$B_\alpha^L = 3,\quad B_\alpha^U = 7$$

$$\text{Then } D(\tilde{A}, \tilde{B}) = \frac{1}{2}[A_\alpha^L + A_\alpha^U - B_\alpha^L - B_\alpha^U]$$

$$= \frac{1}{2}\left[3\frac{1}{2} + 6\frac{1}{2} - 3 - 7\right] = 0$$

$\therefore \quad \tilde{A} \approx \tilde{B}$

3.6. Property

Let $\tilde{A}$, $\tilde{B} \in F$,

i. If $\tilde{A} = \tilde{B}$, Then $\tilde{A} \approx \tilde{B}$

ii. If $\tilde{A} \subset \tilde{B}$, and $\frac{1}{2}$ $[A_\alpha^L + A_\alpha^L] \le \frac{1}{2}$ $[B_\alpha^L + B_\alpha^L]$

Then $\tilde{A} \prec \tilde{B}$

iii. if $\tilde{B} \subset \tilde{A}$ and $\frac{1}{2}$ $[B_\alpha^L + B_\alpha^L] \le \frac{1}{2}$ $[A_\alpha^L + A_\alpha^L]$

Then $\tilde{B} \prec \tilde{A}$

Proof

i. If $\tilde{A} = \tilde{B}$, then for each $\alpha \in [0,1]$, their α–cuts $A_L(\alpha) = B_L(\alpha)$ and $A_U(\alpha) = B_U(\alpha)$, Which implies $A_\alpha^L = B_\alpha^L$ and $A_\alpha^U = B_\alpha^U$

Therefore $D(\tilde{A}, \tilde{B}) = 0 \Rightarrow \tilde{A} \approx \tilde{B}$

ii and iii are obvious.

3.7. Remark

If $\tilde{A} \approx \tilde{B}$, it is not necessary that $\tilde{A} = \tilde{B}$

Therefore, '≈' defined in ordering for fuzzy sets is not equal to '=' defined for fuzzy sets.

3.8. Property

For $\tilde{A}, \tilde{B}, \tilde{C} \in F$,

i. $D(\tilde{A}, \tilde{B}) = -D(\tilde{B}, \tilde{A})$

ii. $D(\tilde{A}, \tilde{B}) + D(\tilde{B}, \tilde{C}) = D(\tilde{A}, \tilde{C})$

iii. $\tilde{A} \approx \tilde{B}$ iff $\tilde{B} \approx \tilde{A}$

iv. $\tilde{A} \approx \tilde{B}$, $\tilde{B} \approx \overline{C} \Rightarrow \tilde{A} \approx \tilde{C}$

Proof

i. It follows from the definition

ii. $D(\tilde{A}, \tilde{B}) + (\tilde{B}, \tilde{C}) = (\tilde{A}, \hat{0}) - D(\tilde{B}, \hat{0}) + D(\tilde{B}, \hat{0}) - D(\tilde{C}, \hat{0})$

$= (\tilde{A}, \hat{0}) - D(\tilde{C}, \hat{0})$

$= D(\tilde{A}, \tilde{C})$

iii. $\tilde{A} \approx \tilde{B} \Leftrightarrow D(\tilde{A}, \tilde{B}) = 0 = -D(\tilde{B}, \tilde{A})$

$\Leftrightarrow \tilde{B} \approx \tilde{A}$, by (i)

iv. $\tilde{A} \approx \tilde{B}$, $\tilde{B} \approx \tilde{C} \Rightarrow D(\tilde{A}, \tilde{B}) = 0$ and $D(\tilde{B}, \tilde{C}) = 0$

from (ii), $D(\tilde{A}, \tilde{B}) + D(\tilde{B}, \tilde{C}) = D(\tilde{A}, \tilde{C})$

$\Rightarrow 0 + 0 = D(\tilde{A}, \tilde{C})$

$\Rightarrow \tilde{A} \approx \tilde{C}$

3.9. Property

In F, the relations $\prec$ and $\approx$ satisfy the three axioms for the order relations, for any $\tilde{A}, \tilde{B}, \tilde{C} \in F$

i. Reflexive law, $\tilde{A} \preceq \tilde{A}$

ii. Anti symmetric law, $\tilde{A} \preceq \tilde{B}$, $\tilde{B} \preceq \tilde{A} \Rightarrow \tilde{A} \approx \tilde{B}$

iii. Transitive law, $\tilde{A} \preceq \tilde{B}$, $\tilde{B} \preceq \tilde{C} \Rightarrow \tilde{A} \approx \tilde{C}$

Proof

i. If follows from the definition

ii. Since $\tilde{A} \preceq \tilde{B}$ and $\tilde{B} \preceq \tilde{A}$, $D(\tilde{B}, \tilde{A}) \ge 0$ and $D(\tilde{B}, \tilde{A}) \le 0$

Therefore $D(\tilde{B}, \tilde{A}) = 0 \Rightarrow \tilde{B} \approx \tilde{A}$ or $\tilde{A} \approx \tilde{B}$

iii. Since $\tilde{A} \preceq \tilde{B}$ and $\tilde{B} \preceq \tilde{C}$, therefore $D(\tilde{B}, \tilde{A}) \ge 0$ and $D(\tilde{C}, \tilde{B}) \ge 0$

By property

$$D(\tilde{C}, \tilde{A}) = D(\tilde{C}, \tilde{B}) + D(\tilde{B}, \tilde{A}) \ge 0 + 0$$

$$\Rightarrow \tilde{A} \le \tilde{C}$$

3.10. Definition

For $\tilde{A}, \tilde{B}, \tilde{C} \in F$, let $d(\tilde{A}, \tilde{B}) = | D(\tilde{A}, \tilde{B}) |$ we call $d(\tilde{A}, \tilde{B})$, the distance between $\tilde{A}$ and $\tilde{B}$.

Then we can show the following axiom

If $\tilde{A}, \tilde{B}, \tilde{C} \in F$

$$d(\tilde{A}, \tilde{B}) + d(\tilde{B}, \tilde{C}) \ge d(\tilde{A}, \tilde{C})$$

Proof

$$d(\tilde{A},\tilde{B}) + d(\tilde{B},\tilde{C}) = |D(\tilde{A},\tilde{B})| + |D(\tilde{B},\tilde{C})|$$
$$\geq |D(\tilde{A},\tilde{B}) + (\tilde{B},\tilde{C})|$$
$$= |D(\tilde{A},\tilde{C})|$$
$$= d(\tilde{A},\tilde{C})$$

4. SOME MORE PROPERTIES

4.1.Theorem

The relation $\prec$ and $\approx$ satisfy the law of trichotomy in F.

Proof

For any $\tilde{A},\tilde{B} \in$ F, by previous definition (3.10), and the property of linear order relation on R, the law of trichotomy holds.

$D(\tilde{A},\tilde{B}) > 0$ or $D(\tilde{A},\tilde{B}) < 0$ or $D(\tilde{A},\tilde{B}) = 0$

$\therefore\ \tilde{B} \prec \tilde{A}$ or $\tilde{A} \prec \tilde{B}$ or $\tilde{A} \approx \tilde{B}$ also holds.

4.2 Property

For $\tilde{A},\tilde{B},\tilde{C}, \in$, F if $|D(\tilde{A},\tilde{B})| < \in$

Then $|D(\tilde{A},\tilde{C}) - D(\tilde{B},\tilde{C})| < \in$

Now $|D(\tilde{A},\tilde{C}) - D(\tilde{B},\tilde{C})|$

$$= |D(\tilde{A},\tilde{C}) - D(\tilde{C},\tilde{B})|$$
$$= |D(\tilde{A},\tilde{B})| < \in$$

4.3.Property

For $\tilde{A},\tilde{B}, \in$, F

i. $\tilde{A}\ (+)\ \tilde{B} = [A_\alpha^L + A_\alpha^L\ ,\ A_\alpha^U + A_\alpha^U]$

ii. $\tilde{A}\ (-)\tilde{B} = [A_\alpha^L - B_\alpha^U\ ,\ A_\alpha^U - B_\alpha^L]$

iii. If $k > 0$, $k\ (\circ)\ \tilde{A} = [kA_\alpha^L\ ,\ kA_\alpha^U]$

iv. If $k < 0$, $k\ (\circ)\ \tilde{A} = [kA_\alpha^U\ ,\ kA_\alpha^L]$

Example

Consider the same example 3.5 (iii)

Two trapezoidal fuzzy numbers :

$$\tilde{A} = (3\ 4\ 6\ 7)\quad \tilde{B} = (2\ 4\ 6\ 8)$$

i. $\tilde{A}\ (+)\ \tilde{B} = \left[6\frac{1}{2}, 13\frac{1}{2}\right]$

ii. $\tilde{A}\ (-)\tilde{B} = \left[-3\frac{1}{2},\ 3\frac{1}{2}\right]$

iii. if $k = 2 > 0$

$$k\ (\circ)\ \tilde{A} = [kA_\alpha^L\ ,\ kA_\alpha^U]$$
$$= \left[2\left(3\frac{1}{2}\right), 2\left(6\frac{1}{2}\right)\right]$$
$$= [\ 7, 13]$$

iv. if $k = -2 < 0$

$$k\ (\circ)\ \tilde{A} = [\ -13, -7]$$

4.4.Theorem

For $\tilde{A},\tilde{B},\tilde{C},\tilde{D} \in$ F, k any scalar

(i) $D(k(\circ)\tilde{A}, k(\circ)\tilde{B}) = k\, D(\tilde{A},\tilde{B})$, $\quad k \neq 0$

(ii) $D(\tilde{A}\ (+)\tilde{B}\ ,\ \tilde{C}(+)\ \tilde{D}) = D(\tilde{A},\tilde{C}) + D(\tilde{B},\tilde{D})$

$$= D(\tilde{A},\tilde{D}) + D(\tilde{B},\tilde{C})$$
$$= D(\tilde{A},\hat{0}) + (\tilde{B},\hat{0}) - D(\tilde{C},\hat{0}) - D(\tilde{D},\hat{0})$$

(iii) $D(\tilde{A}\ (-)\tilde{B}, \tilde{C}\ (-)\tilde{D}) = D(\tilde{A},\tilde{C}) - D(\tilde{B},\tilde{D})$

$$= D(\tilde{A},\tilde{B}) - D(\tilde{C},\tilde{D})$$
$$= D(\tilde{A},\hat{0}) + D(\tilde{D},\hat{0}) - D(\tilde{B},\hat{0}) - D(\tilde{C},\hat{0})$$

Proof (i) By Property, if $k > 0$ then

$k(\circ)\tilde{A} = [k\,A_\alpha^L, k\,A_\alpha^U]$ and

$k(\circ)\tilde{B} = [k\,B_\alpha^L, k\,B_\alpha^U]$

Now

$$D(k(\circ)\tilde{A},\ k(\circ)\tilde{B}) = \frac{1}{2}\,[kA_\alpha^L + k\,A_\alpha^U - k\,B_\alpha^L - k\,B_\alpha^U]$$
$$= \frac{1}{2}\,k\,[A_\alpha^L + A_\alpha^U - B_\alpha^L - B_\alpha^U] = k\,D(\tilde{A},\tilde{B})$$

(ii) By Property

$$\tilde{A}\ (+)\ \tilde{B} = [A_\alpha^L + B_\alpha^L, A_\alpha^U - B_\alpha^U]$$

and $\tilde{C}\ (+)\ \tilde{D} = [C_L^L + D_L^L,\ C_U^U\ - D_U^U]$

Now $D(\tilde{A}\ (+)\tilde{B}, \tilde{C}\ (+)\ \tilde{D})$

$$= \frac{1}{2}[A_\alpha{}^L + B_\alpha{}^L + A_\alpha{}^U + B_\alpha{}^U - C_\alpha{}^L - D_\alpha{}^L - C_\alpha{}^U - D_\alpha{}^U]$$

$$= \frac{1}{2}[A_\alpha{}^L + A_\alpha{}^U - C_\alpha{}^L - C_\alpha{}^U] + \frac{1}{2}[B_\alpha{}^L + B_\alpha{}^U - D_\alpha{}^L - D_\alpha{}^U]$$

$$= D(\tilde{A}, \tilde{C}) + D(\tilde{B}, \tilde{D})$$

$$= \frac{1}{2}[A_\alpha{}^L + A_\alpha{}^U - D_\alpha{}^L - D_\alpha{}^U] + \frac{1}{2}[B_\alpha{}^L + B_\alpha{}^U - C_\alpha{}^L - C_\alpha{}^U]$$

$$= D(\tilde{A}, \tilde{D}) + D(\tilde{B}, \tilde{C})$$

$$= D(\tilde{A}, \hat{0}) - D(\tilde{D}, \hat{0}) + D(\tilde{B}, \hat{0}) - D(\tilde{C}, \hat{0})$$

$$= D(\tilde{A}, \hat{0}) + D(\tilde{B}, \hat{0}) - D(\tilde{C}, \hat{0}) - D(\tilde{D}, \hat{0})$$

(iii) By Property

$(\tilde{A}\ (-)\ \tilde{B}) = [A_\alpha{}^L - B_\alpha{}^U, A_\alpha{}^U - B_\alpha{}^L]$

$(\tilde{C}\ (-)\tilde{D}) = [C_\alpha{}^L - D_\alpha{}^U, C_\alpha{}^U - D_\alpha{}^L]$

Now

$D(\tilde{A}\ (-)\tilde{B}), (\tilde{C}\ (-)\tilde{D})$

$$= \frac{1}{2}[A_\alpha{}^L - B_\alpha{}^U + A_\alpha{}^U - B_\alpha{}^L - C_\alpha{}^L + D_\alpha{}^U - C_\alpha{}^U + D_\alpha{}^L]$$

$$= \frac{1}{2}[A_\alpha{}^L + A_\alpha{}^U - C_\alpha{}^L - C_\alpha{}^U + D_\alpha{}^L + D_\alpha{}^U - B_\alpha{}^L - B_\alpha{}^U]$$

$$= \frac{1}{2}[A_\alpha{}^L + A_\alpha{}^U - C_\alpha{}^L - C_\alpha{}^U] - \frac{1}{2}[B_\alpha{}^L + B_\alpha{}^U - D_\alpha{}^L - D_\alpha{}^U]$$

$$= D[\tilde{A}, \tilde{C}] - D[\tilde{B}, \tilde{D}]$$

Also LHS

$$= \frac{1}{2}[A_\alpha{}^L + A_\alpha{}^U - B_\alpha{}^L + B_\alpha{}^U] - \frac{1}{2}[C_\alpha{}^L + C_\alpha{}^U - D_\alpha{}^L - D_\alpha{}^U]$$

$$= D[\tilde{A}, \tilde{B}] - D[\tilde{C}, \tilde{D}]$$

$$= D(\tilde{A}, \tilde{0}) + D(\tilde{D}, \tilde{0}) - D(\tilde{B}, \tilde{0}) - D(\tilde{C}, \tilde{0})$$

4.5. Theorem

Let $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D} \in F$

(i) If $\tilde{B} \prec \tilde{A}$, Then $\tilde{B}\ (+)\tilde{C} \prec \tilde{A}\ (+)\tilde{C}$

(ii) If $\tilde{C} \prec \tilde{A}$ and $\tilde{D} \prec \tilde{B}$ Then $\tilde{C}\ (+)\tilde{D} \prec \tilde{A}\ (+)\ \tilde{B}$

(iii) If $\tilde{B} \prec \tilde{A}$, Then $\tilde{B}\ (-)\tilde{C} \prec \tilde{A}\ (-)\tilde{C}$

(iv) If $\tilde{A} \prec \tilde{C}$, $\tilde{B} \prec \tilde{D}$, Then $\tilde{A}\ (-)\tilde{D} \prec \tilde{C}\ (-)\ \tilde{B}$ and $\tilde{B}\ (-)\tilde{C} \prec \tilde{D}\ (-)\ \tilde{A}$

Proof

(i) We have

$D(\tilde{A}\ (+)\tilde{C}, \tilde{B}\ (+)\ \tilde{C}) = D(\tilde{A}, \tilde{B}) + D(\tilde{C}, \tilde{C}) > 0,$

Since $D(\tilde{C}, \tilde{C}) = 0$, and $\tilde{B} < \tilde{A}$, $D(\tilde{A}, \tilde{B}) > 0$

$$\Rightarrow \tilde{B}\ (+)\tilde{C} \prec \tilde{A}\ (+)\tilde{C}$$

(ii) We have

$D(\tilde{A}\ (+)\tilde{B}, \tilde{C}\ (+)\tilde{D}) = D(\tilde{A}, \tilde{C}) + D(\tilde{B}, \tilde{D}) > 0$

Since $\tilde{C} \prec \tilde{A} \Rightarrow D(\tilde{A}, \tilde{C}) > 0$ and $\tilde{D} \prec \tilde{B} \Rightarrow D(\tilde{B}, \tilde{D}) > 0$

$$\Rightarrow (\tilde{C}\ (+)\tilde{D}) \prec (\tilde{A}\ (+)\tilde{B})$$

(iii) We have

$D(\tilde{A}\ (-)\tilde{C}, \tilde{B}\ (-)\tilde{C}) = D(\tilde{A}, \tilde{B}) - D(\tilde{C}, \tilde{C}) > 0,$

Since $D(\tilde{C}, \tilde{C}) = 0$ and $\tilde{B} \prec \tilde{A} \Rightarrow D(\tilde{A}, \tilde{B}) > 0$

$$\Rightarrow \tilde{B}\ (-)\tilde{C} \prec \tilde{A}\ (-)\tilde{C}$$

(iv) If $\tilde{A} \prec \tilde{C}$, $\tilde{B} \prec \tilde{D}$

$-D(\tilde{A}, \tilde{C}) = D(\tilde{C}, \tilde{A}) > 0$, and $-D(\tilde{B}, \tilde{D}) = D(\tilde{D}, \tilde{B}) > 0$

We have

$D(\tilde{C}\ (-)\tilde{B}, \tilde{A}\ (-)\tilde{D}) = D(\tilde{C}, \tilde{A}) - D(\tilde{B}, \tilde{D}) > 0$

$$\Rightarrow (\tilde{A}\ (-)\tilde{D}) \prec (\tilde{C}\ (-)\tilde{B})$$

Also,

$D(\tilde{D}\ (-)\tilde{A}, \tilde{B}\ (-)\ \tilde{C}) = D(\tilde{D}, \tilde{B}) - D(\tilde{A}, \tilde{C}) > 0$

$$\Rightarrow \tilde{B}\ (-)\tilde{C} \prec \tilde{D}\ (-)\tilde{A}$$

5. ILLUSTRATIONS

5.1. Example Let $\tilde{A} = (1, 2, 3, 4)$,

$\tilde{B} = (2, 4, 6, 8)$

$\tilde{C} = (2, 5, 7, 10)$

$\tilde{D} = (3, \quad 6, \quad 9, \quad 12)$ be trapezoidal fuzzy numbers.

The Close Interval distance of fuzzy numbers are :

$$D(\tilde{A}, \hat{0}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4) = \frac{1+2+3+4}{4} = 2.5$$

$D(\tilde{B}, \hat{0}) = 5$, $D(\tilde{C}, \hat{0}) = 6$, $D(\tilde{D}, \hat{0}) = 7.5$

$D(\tilde{C}, \hat{0}) = 6$

$D(\tilde{D}, \hat{0}) = 7.5$

$\therefore D(\tilde{D}, \hat{0}) > D(\tilde{C}, \hat{0}) > D(\tilde{B}, \hat{0}) > D(\tilde{A}, \hat{0})$,

$\tilde{A} \prec \tilde{B} \prec \tilde{C} \prec \tilde{D}$.

The α–cuts of fuzzy numbers are :

$A_L(\alpha) = 1 + \alpha$, $A_U(\alpha) = 4 - \alpha$

$B_L(\alpha) = 2+2\alpha$, $B_U(\alpha) = 8 - 2\alpha$

$C_L(\alpha) = 2 + 3\alpha$, $C_U(\alpha) = 10 - 3\alpha$

$D_L(\alpha) = 3 + 3\alpha$, $D_U(\alpha) = 12 - 3\alpha$

The close interval approximation of trapezoidal numbers are

$[A] = [1.5,\ 3.5]$, $[B] = [3,7]$, $[C] = [3.5,\ 8.5]$ and $[D] = [4.5,\ 10.5]$

Now $\tilde{A}\ (+)\ \tilde{B} = (3,6,9,12)$

$\tilde{C}\ (+)\ \tilde{D} = (5,11,16,22)$

$$D(\tilde{A}\ (+)\ \tilde{B}, \tilde{C}\ (+)\ \tilde{D})$$

$$= \frac{1}{2}[3+1.5 + 12-1.5 - (5+3) - (22-3)] = \frac{-12}{2} \quad (1)$$

$$D(\tilde{A}, \tilde{C}) = \frac{1}{2}[1.5 + 3.5 - 3.5 - 8.5] = \frac{-7}{2} \quad (2)$$

$$D(\tilde{B}, \tilde{D}) = \frac{1}{2}[3+7 - 4.5 - 10.5] = \frac{-5}{2} \quad (3)$$

$$D(\tilde{A}, \tilde{D}) = \frac{1}{2}[1.5 + 3.5 - 4.5 - 10.5] = -5 \quad (4)$$

$$D(\tilde{B}, \tilde{C}) = \frac{1}{2}[10 - 12] = \frac{-2}{2} = -1 \quad (5)$$

Using (1) – (5) we have

$$D(\tilde{A}\ (+)\ \tilde{B}, \tilde{C}\ (+)\ \tilde{D}) = D(\tilde{A}, \tilde{C}) + D(\tilde{B}, \tilde{D})$$

$$= D(\tilde{A}, \tilde{D}) + D(\tilde{B}, \tilde{C})$$

Also, $\tilde{A}\ (-)\ \tilde{B} = (1-8, 2-6, 3-4, 4-2) = (-7, -4, -1, 2)$ $\tilde{C}\ (-)\ \tilde{D} = (2-12, 5-9, 7-6, 10-3) = (-10, -4, 1, 7)$

$$D(\tilde{A}\ (-)\ \tilde{B}, \tilde{C}\ (-)\ \tilde{D})$$

$$= \frac{1}{2}\left[-7 + 3\left(\frac{1}{2}\right) + 2 - 3\left(\frac{1}{2}\right) - (-10+3) - (7-3)\right]$$

$$= -1 \quad (6)$$

Now

$$D(\tilde{A}, \tilde{C}) - D(\tilde{B}, \tilde{D}) = \frac{-7}{2} + \frac{5}{2} = \frac{-2}{2} = -1 \quad (7)$$

Also

$$D(\tilde{A}, \tilde{B}) = \frac{1}{2}[5 - 10] = \frac{-5}{2}$$

$$D(\tilde{C}, \tilde{D}) = \frac{1}{2}[12 - 15] = \frac{-3}{2}$$

$$\text{Now } D(\tilde{A}, \tilde{B}) - D(\tilde{C}, \tilde{D}) = \frac{-5}{2} + \frac{-3}{2} = -4 \quad (8)$$

From (6), (7) and (8)

$$D(\tilde{A}\ (-)\ \tilde{B}, \tilde{C}\ (-)\ \tilde{D}) = D(\tilde{A}, \tilde{C}) - D(\tilde{B}, \tilde{D})$$

$$= D(\tilde{A}, \tilde{B}) - D(\tilde{C}, \tilde{D})$$

Hence proved.

5.2.Example

We shall now compare the different fuzzy numbers, such as triangular, ($\tilde{A}$), Trapezoidal ($\tilde{B}$) and piecewise quadratic ($\tilde{C}$) fuzzy numbers using the earlier notions.

$\tilde{A} = (2 \quad 4 \quad 6)$

$\tilde{B} = (3 \quad 4 \quad 6 \quad 7)$

$\tilde{C} = (1 \quad 3 \quad 4 \quad 5 \quad 7)$

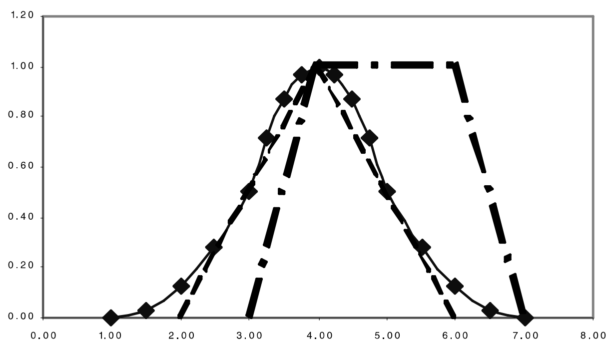
By using our method, we have the following results.

$$d(\tilde{A}, \hat{0}) = \frac{1}{4}\,(2 + 2(4) + 6) = 4$$

$$d(\tilde{B}, \hat{0}) = \frac{1}{4}\,(3 + 4 + 6 + 7) = \frac{20}{4} = 5$$

$$d(\tilde{C}, \hat{0}) = \frac{1}{4}(1 + 2(4) + 7) = \frac{16}{4} = 4$$

$\therefore$ By definition, $\tilde{A} \approx \tilde{C} \prec \tilde{B}$.



6. REFERENCES

Baldwin, J.F. and Guild, N.C.F. 1979. Comparison of fuzzy numbers on the same decision space. *Fuzzy Set Syst.* 2: 213-233.

Gonzalez, A. 1990. A study of the ranking function approach through mean values. *Fuzzy Set and Syst.* 35: 29-41.

Kaufmann, A. and Gupta, M.M. 1991. *Introduction to Fuzzy Arithmetic Theory and Application.* Van Nostrand Rein hold, New York.

Nagoorgani, A. and Stephen Dinagar, D. 2004. A new representation of piecewise quadratic fuzzy number using close interval approximations. *Bull. Pure Appl. Sci.* 23E: 225– 233.

Saade, J. and Schwarzlander, H. 1992. Ordering fuzzy sets over the real time an approach based on decision making under uncertainty. *Fuzzy Set Sys.* 50: 237 – 246.

Yager, R.R. 1981. A procedure for ordering fuzzy sub sets of the unit interval. *Inform. Sci.* 24: 143-161.